Thursday 10/17/19
1. Grab Notes & Calc.
2. Take out HW, warm-up
3. Rational Root Thm. Notes
4. Break
5. Roots/Zeros of Polyn. Notes
6. HW/Practice
**Topic:** Rational Root Theorem

**What am I learning today?**

### Warm-Up:
Prove the zeros of the polynomial with synthetic division. Then find the remaining zero(s) using the result.

1. \( f(x) = 2x^2 - 3x^2 - 32x - 16; x = \frac{1}{2}, x = -5 \)

\[
\begin{array}{c|cc|c}
 & 2 & -3 & -32 & -16 \\
\hline
\frac{1}{2} & & & & \\
\hline
& 1 & -\frac{5}{2} & -15 \\
\hline
& 2 & -5 & -30 & -10 \\
\hline
& & \frac{1}{2} & 10 & 10 \\
\hline
\end{array}
\]

\( x = -\frac{3}{2} \)

2. \( x = -\frac{5}{2} \)

3. \( x = -3 \)

### Vocabulary

**Rational Root Theorem**

\( \pm \frac{c}{l} \)

The ____________ provides a complete list of all possible rational roots of a polynomial equation.

To find all possible rational roots, do the following:

\[ \pm \frac{c}{l} = \pm \text{factors of the constant term} \]

\[ \pm \frac{1}{l} = \pm \text{factors of the leading coefficient} \]

**Ex. 1:** Find/list all the possible rational roots of the following polynomials.

a. \( f(x) = -2x^2 + 3x + 15x - 40 \)

\( C(40): 1, 40, 10, 4 \)

\( L(8): 1, 8 \)

\( \pm \frac{c}{l} \) (PRR):

\( \pm 1, \pm \frac{1}{2}, \pm 5, \pm 4, \pm 10, \pm \frac{20}{2}, \ldots \)

b. \( f(x) = -15x^4 + 3x^3 - 9x^2 - x + 24 \)

\( C(24): 1, 24, 6, 4, 2, 12, 8 \)

\( L(5): 1, 5, 3, 5 \)

\( \pm \frac{c}{l} \) (PRR):

\( \pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{2}, \pm \frac{1}{5}, \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 9, \pm 12, \pm 24, \pm \frac{24}{5} \)

### Recall

**Writing Zeros as Factors**

Recall from a previous unit how we wrote zeros as factors so we could write a polynomial in factored form.

**Ex. 2:** Write the following zeros/roots/x-intercepts as a polynomial in factored form.

a. \( x = -\frac{2}{3}, x = -4, x = 3 \)

\( 3x - \frac{2}{3} \)

\( 3x^2 + 14x - 12 = (3x - 2)(x + 4)(x - 3) \)

b. \( x = 0, x = -\frac{5}{2}, x = -2, x = -1 \)

\( x = \frac{1}{2} \)

\( x = -2, x = 2 \)

\( x + 2 = 0, x = 2, x = -2 \)

\( (2x + 5)(x + 2)(x - 1) \)
**Topic:** Rational Root Theorem

**Steps**

Steps to find the zeros of a polynomial and use the zeros to write the polynomial in factored form:

1. Find/list all the possible rational roots \( \pm \frac{p}{q} \)
2. Use the calculator to find the rational roots (see steps below)
3. Prove the zeros from the calculator are zeros/roots using synthetic division.
4. Solve for any missing zeros (using the last line from synthetic division).
5. Use found zeros to write in factored form.

**Calculator Steps:**

1. Press \( \text{Table} \), select \( \text{Edit Function} \), enter a functions and press \( \text{Enter} \).
2. Select \( \text{Table Start} \) and \( \text{Table Step} \) values, then hit \( \text{Calc} \).

**Example**

Ex. 3: List all possible rational roots. Then use a calculator, synthetic division and factoring to find all zeros & write in factored form (no fractions).

\[
f(x) = x^4 - x^2 - 17x - 15
\]

PRR: \( \pm 1, \pm 3, \pm 5, \pm 15 \)

Zeros: \( x = -1, x = 3, x = -5 \)

Factored Form:

\[
f(x) = (x+1)(x-3)(x+5)
\]

**Example**

Ex. 4: List all possible rational roots. Then use a calculator, synthetic division and factoring to find all zeros & write in factored form (no fractions).

\[
f(x) = 2x^4 + 11x^3 + 13x^2 - 11x - 15
\]

PRR: \( \pm 1, \pm \frac{1}{2}, \pm 3, \pm \frac{3}{2}, \pm 5, \pm \frac{5}{2} \)

Zeros: \( x = 1, x = -1, x = 3, x = -\frac{5}{2} \)

Factored Form:

\[
f(x) = (x-1)(x+1)(x+3)(2x+5)
\]

**You Try**

Ex. 5: List all possible rational roots. Then use a calculator, synthetic division and factoring to find all zeros & write in factored form (no fractions).

\[
f(x) = 5x^3 + 2x^2 - 45x - 18
\]

PRR:

Zeros: \( \underline{ \underline{ \underline{ } } } \)

Factored Form:

\[
f(x) = \underline{ \underline{ \underline{ } } } \]
Friday 10/18/19

1. Grab Calc.
2. Start Warm-Up (on 2nd sheet of notes from yesterday)
3. HW & Warm-Up
4. Finish Zeros to Factors Notes
**Topic:** Roots/Zeros of Polynomials to Factors

**What am I learning today?**

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**Warm-Up**

Solve the following polynomials by factoring:

1. \( f(x) = 3x^2 - 10x + 8 \)
   - \( (3x - 2)(x - 4) \)
   - \( 2(x - 2)(x - 4) \)
   - \( x = \frac{2}{3}, x = 4 \)

2. \( f(x) = 3x^2 + 12x \)
   - \( 3x(x + 4) = 0 \)
   - \( 3x = 0, x + 4 = 0 \)
   - \( x = 0, x = -4 \)

3. \( f(x) = 2x^2 - 98 \)
   - \( 2(x^2 - 49) \)
   - \( 2(x + 7)(x - 7) \)
   - \( x + 7 = 0, x - 7 = 0 \)
   - \( x = -7, x = 7 \)

**Vocabulary**

**Roots** & **Zeros** are solutions to a function. When a root/zero is plugged into a variable, it makes the function equal to 0.

To find the roots/zeros of a function, factor the polynomial COMPLETELY, then set each factor equal to zero and solve (like in the warm-up).

However, if you’re given the roots/zeros and are asked to create the polynomial, then you have to work backwards. Follow these steps:

1. Write down the **roots/zeros**.
2. Write the corresponding factors, by getting each equal to zero.
3. Multiply out the factors (Factored Form or FOIL method).
4. Use "f(x)" notation when writing your final polynomial.

**Example:**

Given the following roots/zeros, write the corresponding polynomial in factored & polynomial form:

a. \( x = 2 \) and \( x = -1/3 \)
   - \( f(x) = (x - 2)(3x + 1) \)
   - \( f(x) = 3x^2 + 5x - 2 \)

b. \( x = 3 \) and \( x = -3 \)
   - \( f(x) = (x - 3)(x + 3) \)
   - \( f(x) = x^2 - 9 \)
Topic: Roots/Zeros of Polynomials to Factors

### You Try

Given the following roots/zeros, write the corresponding polynomial in factored & polynomial form.

1. \( x = -\frac{2}{3} \) and \( x = 1 \)

\[
\left(3x+\frac{2}{3}x+1\right) \\
\left(x-1\right)
\]

\( f(x) = 3x^2 + 1x - 2 \)

2. \( x = 0 \) and \( x = 5 \)

\( f(x) = x^2 - 5x \)

3. \( x = -1 \) and \( x = \frac{1}{2} \)

\( f(x) = x^2 - 2x - \frac{3}{2} \)

\[
\frac{f(-1)}{x+1} \\
\frac{f\left(\frac{1}{2}\right)}{x-\frac{1}{2}}
\]

### Vocabulary

Complex Roots/Zeros

- \( -4i \)
- \( 1+5i \)
- \( 4i \)
- \( 1-5i \)

Complex Conjugates

- \( -4i \)
- \( 1+5i \)
- \( 4i \)
- \( 1-5i \)

Solutions are not always \( \text{real} \) numbers. Sometimes we get solutions that are \( \text{im} \)aginary (or \( \text{imaginary} \) numbers). Remember, these are solutions that include the letter \( "i" \).

****IMPORTANT****

If a complex number is a root/zero, then so is its \( \text{Conjugate} \).

(Remember, the complex conjugate of \( a + bi \) is \( a - bi \).)

For example: if \( x = 4i \) is a root/zero, then so is its conjugate, \( x = -4i \).

### Examples

Example: Given the following roots/zeros, write the corresponding polynomial in factored & polynomial form.

a. \( x = 3i \)

\[
(x-3i)(x+3i) \\
x^2 + 9
\]

\( f(x) = x^2 + 9 \)

b. \( x = 2 \), \( x = 5i \)

\[
(x-2)(x-5i)(x+5i) \\
(x-2)(x^2 + 25x - 25)
\]

\( f(x) = x^2 + 25x - 25 \)

### You Try

Given the following roots/zeros, write the corresponding polynomial in factored & polynomial form.

a. \( x = -2i \)

b. \( x = 0 \), \( x = -i \)

### Summary

Summarize the lesson in your own words.