Thursday 10/17/19

1. Grab Notes & Calc.
2. Take out HW, warm-up
3. Rational Root Thm. Notes
4. Break
5. Roots/Zeros of Polyn. Notes
6. HW/Practice

Topic: Rational Root Theorem **Name:** _____

What am I learning today?

Warm-Up:

Prove the zeros of the polynomial with synthetic division. Then find the remaining zero(s) using the result.

1. $f(x) = 2x^3 - 3x^2 - 32x - 15;\ x = -\frac{1}{2}, x = 5$

$-\frac{1}{2}$	2	-3	-32	-15
		-1	2	15
5	2	-4	-30	0
		10	30	
	2	6	0	
	x	c		

$x = -3$

$2x + 6 = 0$

$\frac{2x}{2} = \frac{-6}{2}$

$\boxed{x = -3}$

Vocabulary

Rational Root Theorem

$\pm \frac{c}{L}$

The rational root thm. provides a complete list of all possible rational roots of a polynomial equation.

To find all possible rational roots do the following:

$$\pm\frac{C}{L} = \pm\frac{\text{factors of the constant term}}{\text{factors of the leading coefficient}}$$

Ex. 1: Find/list all the possible rational roots of the following polynomials.

a. $f(x) = 3x^3 - 3x^2 + 15x - 40$

C(40): 1, 40, 10, 4, 8, 5, 2, 20

L(3): 1, 3

$\pm\frac{c}{L}$ (PRR): $\pm 1, \pm\frac{1}{3}, \pm 40, \pm\frac{40}{3}, \pm 10, \pm\frac{10}{3}$ .....

b. $f(x) = -15x^4 + 3x^3 - 9x^2 - x + 24$

C(24): 1, 24, 6, 4, 2, 12, 3, 8

L(15): 1, 15, 3, 5

$\pm\frac{c}{L}$ (PRR): $\pm 1, \pm\frac{1}{15}, \pm\frac{1}{3}, \pm\frac{1}{5}, \pm 24, \pm\frac{8}{5}, \pm 8, \pm\frac{24}{5}$ ...

Recall

Writing Zeros as Factors

Recall from a previous unit how we wrote zeros as factors so we could write a polynomial in factored form.

Ex. 2: Write the following zeros/roots/x-intercepts as a polynomial in factored form.

a. $x = \frac{2}{3}, x = -4, x = 3$

$3 \cdot x = \frac{2}{3} \cdot 3$ $x = -4$ $x = 3$

$3x = 2$ $x + 4 = 0$ $x - 3 = 0$

$3x - 2 = 0$

$(3x - 2)(x + 4)(x - 3)$

b. $x = 0, x = -\frac{5}{2}, x = -2, x = 1$

x $2 \cdot x = -\frac{5}{2} \cdot 2$ $x = -2$ $x = 1$

$2x = -5$ $x + 2 = 0$ $x - 1 = 0$

$2x + 5 = 0$

$= (x)(2x + 5)(x + 2)(x - 1)$

Topic: Rational Root Theorem **Date**: _____

Steps

Finding All Zeros and Writing in Factored Form

Steps to find the zeros of a polynomial and use the zeros to write the polynomial in factored form:

1. Find/list all the possible rational roots ($\pm\frac{c}{L}$)
2. Use the calculator to find the rational roots (see steps below)
3. Prove the zeros from the calculator are zeros/roots using synthetic division.
4. Solve for any missing zeros (using the last line from synthetic division).
5. Use found zeros to write in factored form.

CALCULATOR STEPS:
1. Press table , select **Edit Function,** enter a functions and press enter .
2. Select **Table Start** and **Table Step** values, then hit CALC .

Example

Ex. 3: List all possible rational roots. Then use a calculator, synthetic division and factoring to find all zeros & write in factored form (no fractions).

C(15): 1,3,5,15
L(1): 1

$f(x) = \overset{L}{x^3} - x^2 - 17x - \overset{C}{15}$

PRR: ±1, ±3, ±5, ±15

(from calc)

Zeros: x=-1, x=-3, x=5

x=-1
x+1=0
x=-3
x+3=0
x=5
x-5=0

Factored Form:

f(x) = (x+1)(x+3)(x-5)

-1	1	-1	-17	-15
	↓	-1	2	15
-3	1	-2	-15	0
	↓	-3	15	
5	1	-5	0	
	↓	5		
	1	0		

Example

Ex. 4: List all possible rational roots. Then use a calculator, synthetic division and factoring to find all zeros & write in factored form (no fractions).

C(15): 1,3,5,15
L(2): 1,2
x=1 → x-1=0
x=-1 → x+1=0
x=-3 → x+3=0
2x = -5/2
2x=-5
2x+5=0

$f(x) = \overset{L}{2x^4} + 11x^3 + 13x^2 - 11x - \overset{C}{15}$

PRR: ±1, ±1/2, ±3, ±3/2, ±5, ±5/2, ±15, ±15/2

Zeros: x=1, x=-1, x=-3, x=-5/2

Factored Form:

f(x) = (x-1)(x+1)(x+3)(2x+5)

1	2	11	13	-11	-15
	↓	2	13	26	15
-1	2	13	26	15	0
	↓	-2	-11	-15	
-3	2	11	15	0	
	↓	-6	-15		
	2	5	0		
	x	c			

2x+5=0
2x=-5
x=-5/2

You Try

Ex. 5: List all possible rational roots. Then use a calculator, synthetic division and factoring to find all zeros & write in factored form (no fractions).

$f(x) = 5x^3 + 2x^2 - 45x - 18$

PRR:

Zeros: _____

Factored Form:

f(x) = _____

Friday 10/18/19

1. Grab Calc.
2. Start Warm-Up (on 2nd sheet of notes from yesterday)
3. HW & Warm-Up
4. Finish Zeros to Factors Notes

Topic: Roots/Zeros of Polynomials to Factors **Name**: _____

What am I learning today?

Warm-Up

M 24 | A -10
-6, -4

Solve the following polynomials by factoring:

1. $f(x) = 3x^2 - 10x + 8$

$(3x^2 - 6x - 4x + 8)$

$3x(x-2) - 4(x-2)$

$(3x-4)(x-2)$

$3x - 4 = 0$ $\quad$ $x - 2 = 0$

$3x = 4$ $\quad$ $\boxed{x = 2}$

$\boxed{x = \frac{4}{3}}$

2. $f(x) = 3x^2 + 12x$

$\frac{3x}{3x} \quad \frac{3x}{3x}$

$3x(x+4) = 0$

$3x = 0$ $\quad$ $x + 4 = 0$

$\boxed{x = 0}$ $\quad$ $\boxed{x = -4}$

3. $f(x) = 2x^2 - 98$

$2(x^2 - 49)$

$2(x+7)(x-7)$

$x + 7 = 0$ $\quad$ $x - 7 = 0$

$\boxed{x = -7}$ $\quad$ $\boxed{x = 7}$

Vocabulary

Roots/Zeros

Solutions
x-Intercepts

Roots & Zeros are solutions to a function. When a root/zero is plugged into a variable, it makes the function equal to 0.

To find the roots/zeros of a function, factor the polynomial COMPLETELY, then set each factor equal to zero and solve (like in the warm-up).

However, if you're given the roots/zeros and are asked to create the polynomial, then you have to work backwards. Follow these steps:

1. Write down the zeros.
2. Write the corresponding factors by getting each equal to zero. (FACTORED FORM)
3. Multiply out the factors (FOIL or DISTRIBUTE).
4. Use "$f(x)$" notation when writing your final polynomial.

Example:

Given the following roots/zeros, write the corresponding polynomial in factored & polynomial form:

a. **x = 2 and x = -1/3**

$3 \cdot x = \frac{-1}{3} \cdot 3$

$3x = -1$

$3x + 1 = 0$

$(x-2)(3x+1)$

FACTORED FORM:

$\boxed{f(x) = (x-2)(3x+1)}$

POLYNOMIAL FORM:

$f(x) = (x-2)(3x+1)$

$= 3x^2 + x - 6x - 2$

$\boxed{f(x) = 3x^2 - 5x - 2}$

b. **x = 3 and x = -3**

FACTORED FORM:

$\boxed{f(x) = (x-3)(x+3)}$

POLYNOMIAL FORM:

$f(x) = (x-3)(x+3)$

$= x^2 + \cancel{3x} - \cancel{3x} - 9$

$\boxed{f(x) = x^2 - 9}$

Topic: Roots/Zeros of Polynomials to Factors **Date**: _____

You Try

Given the following roots/zeros, write the corresponding polynomial in **factored** & **polynomial** form

1. x = -2/3 and x = 1
2. x = 0 and x = 5
3. x = 4, x = -1 and x = 1/2

$3x = -\frac{2}{3} \cdot 3$

$3x = -2$

$+2 \quad +2$

$3x+2$

$\boxed{(3x+2)(x+1)}$

$(3x+2)(x-1)$

$3x^2-3x+2x-2$

$f(x)=3x^2-1x-2$

$(x) \quad (x-5)$

$f(x)=(x)(x-5)$

$f(x)=x^2-5x$

Vocabulary

Complex Roots/Zeros

Complex Conjugates

$-4i \rightarrow 4i$

$1+3i \rightarrow 1-3i$

Solutions are not always real numbers. Sometimes we get solutions that are imaginary numbers (or "i" numbers). Remember, these are solutions that include the letter "i".

******IMPORTANT******

If a complex number is a root/zero, then so is it's conjugate

(Remember, the complex conjugate of $a + bi$ is $a-bi$)

For example: if $x = 4i$ is a root/zero, then so is it's conjugate, $x = -4i$

Examples

$i^2 = -1$

Example: Given the following roots/zeros, write the corresponding polynomial in factored & polynomial form.

a. $x = 3i$

$x=-3i$

$(x-3i)(x+3i)$

$x^2+\cancel{3xi}-\cancel{3xi}-9i^2$ (-1)

$\boxed{f(x)=x^2+9}$

b. $x = 2$, $x = 5i$

$x=-5i$

$f(x)=(x-2)(x-5i)(x+5i)$

$=(x-2)(x^2+\cancel{5xi}-\cancel{5xi}-25i^2)$ (-1)

$=(x-2)(x^2+25)$

$=x^3+25x-2x^2-50$

You Try

Given the following roots/zeros, write the corresponding polynomial in factored & polynomial form.

a. $x = -2i$

b. $x = 0$, $x = -i$

Summary

Summarize the lesson in your own words