

Unit 1 Review Problems Algebra II

Name

KEY

Simplify each expression:

1. $\sqrt{48}$

$4\sqrt{3}$

2. $\sqrt{75}$

$5\sqrt{3}$

3. $\sqrt{162}$

$9\sqrt{2}$

4. $2x^3 \cdot 3x^7$

$6x^{10}$

5. $\frac{4x^5y^3}{3x^2y}$

$\frac{4x^3y^2}{3}$

6. $(2ab^2c^3)^3$

$8a^3b^6c^9$

7. $\sqrt{-18}$

$3i\sqrt{2}$

8. $\sqrt{-49}$

$7i$

9. $(4i - 3) + (2 - i)$

$-1 + 3i$

10. $(3 + 2i) - (5 - 6i)$

$-2 + 8i$

11. $2i(6 - i)$

$2 + 12i$

12. $(2i + 3)(3i - 5)$

$-21 - i$

Factor each polynomial:

13) $-2xy - 2x$

$-2x(y + 1)$

14) $2x^3 + 10x$

$2x(x^2 + 5)$

15) $x^2 + 10x + 25$

$(x + 5)(x + 5)$

16) $x^2 - 3x - 54$

$(x - 9)(x + 6)$

11) $x^2 - 8x + 12$

$(x - 6)(x - 2)$

12) $x^2 - 25$

$(x + 5)(x - 5)$

13) $3x^2 - 14x - 5$

$(3x^2 - 15x) + 1(x - 5)$

$\frac{-15}{-15, 1}$

$3x(x - 5) + 1(x - 5)$

$(3x + 1)(x - 5)$

14) $4x^2 + 8x + 3$

$(4x^2 + 6x) + (2x + 3)$

$\frac{12}{6, 2}$

$2x(2x + 3) + 1(2x + 3)$

$(2x + 1)(2x + 3)$

15) $x^3 + 343$

$= (x + 7)(x^2 - 14x + 49)$

Solve each quadratic equation by factoring:

16) $x^2 - 25 = 0$

$$x = 5, -5$$

17) $x^2 - 2x - 24 = 0$
 $(x-6)(x+4)$

$$x = 6, -4$$

18) $4x^2 - 15 = -7x$

$$x = -3, \frac{5}{4}$$

$$4x^2 + 7x - 15$$
$$(4x^2 + 12x) - 5(x-15)$$
$$4x(x+3) - 5(x+3)$$

$$\frac{-7 \pm \sqrt{49 - 4(4)(-15)}}{2(4)}$$
$$\frac{-7 \pm \sqrt{49 + 240}}{8}$$
$$\frac{-7 \pm \sqrt{289}}{8}$$
$$\frac{-7 \pm 17}{8}$$
$$\frac{-7+17}{8} = \frac{10}{8} = \frac{5}{4}$$
$$\frac{-7-17}{8} = \frac{-24}{8} = -3$$

Solve by quadratic formula.

19) $2x^2 - x - 6 = 0$

$$\frac{1 \pm \sqrt{49}}{4} = 2, -\frac{3}{2}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

20) $x^2 + 13x + 7 = 0$

$$\frac{-13 \pm \sqrt{141}}{2} =$$

- Adding polynomials – combine like terms

$$\text{Ex. } (3x^3 + x^2 - 5) + (4x^3 + 7x + 8) = \underline{7x^3 + x^2 + 7x + 3}$$

- Subtracting polynomials – distribute negative and combine like terms

$$\text{Ex. } (4x^3 + 2x^2 - 5x + 2) - (2x^3 + 3x + 8) = 4x^3 + 2x^2 - 5x + 2 - 2x^3 - 3x - 8 = \underline{2x^3 + 2x^2 - 8x - 8}$$

- Multiplying Polynomials

Distribute all terms to all terms. Multiply coefficients and add exponents.

$$\text{Ex. } 2x(x - 3) = \underline{2x^2 - 6x}$$

$$\text{Ex. } (x - 3)(x + 6) = x^2 + 6x - 3x - 18 = \underline{x^2 + 3x - 18}$$

$$\text{Ex. } (x - 1)(2x^2 + 2x - 7) = 2x^3 + 2x^2 - 7x - 2x^2 - 2x + 7 = \underline{2x^3 - 9x + 7}$$

- Dividing Polynomials

You will only have to do synthetic division! We know how to do synthetic division.

- Combination with a value

Plug the number in to each function and then perform the given operation (add, subtract, multiply or divide)

- Composition

$f(g(x))$ means to plug $g(x)$ into $f(x)$

If there's a number plug it in to the inside function first and then take the new number and plug it into the outside function.

- Inverse

1. switch x and y

2. solve for y (get the y by itself)

Add or subtract.

$$1. (5x + 2) + (x - 3)$$

$$6x - 1$$

$$2. (x^2 - 5) - (x + 4)$$

$$x^2 - x - 9$$

$$3. (5x^2 - 6x + 10) - (9x^2 - 4x + 3)$$

$$-4x^2 - 2x + 7$$

$$4. (5x^2 - 15x - 13) + (9x^2 - 4x - 6)$$

$$14x^2 - 19x - 19$$

$$5. (x^3 + 2x^2 - 7x + 4) + (4x^3 - 5x^2 - 2x - 8)$$

$$5x^3 - 3x^2 - 9x - 4$$

$$6. (x^3 - 3x^2 + 4) - (2x^2 - 8x + 6)$$

$$x^3 - 5x^2 + 8x - 2$$

7. $(x + 2)(x - 5)$

$$x^2 - 3x - 10$$

8. $2x(8x - 5)$

$$16x^2 - 10x$$

9. $(x^2 - 3)(2x + 1)$

$$2x^3 + x^2 - 6x - 3$$

10. $(x + 1)(2x^2 - 5x - 8)$

$$2x^3 - 5x^2 - 8x + 2x^2 - 5x - 8$$

$$\boxed{2x^3 - 3x^2 - 13x - 8}$$

Divide using synthetic division. (remember 0's when you have missing factors)

11. $(12x^3 - 11x^2 + 7x + 14) \div (x + 3)$

$$\begin{array}{r|rrrr} -3 & 12 & -11 & 7 & 14 \\ & \downarrow & -36 & 141 & -444 \\ \hline & 12 & -47 & 148 & -430 \end{array}$$

$$\boxed{12x^2 - 47x + 148 - \frac{430}{x+3}}$$

12. $(6x^5 - 33x^4 + 150x^2 - 96) \div (x - 4)$

$$\begin{array}{r|rrrrrrr} 4 & 6 & -33 & 0 & 150 & 0 & -96 \\ & \downarrow & 24 & -36 & -144 & 24 & 96 \\ \hline & 6 & -9 & -36 & 6 & 24 & 0 \end{array}$$

$$\boxed{6x^4 - 9x^3 - 36x^2 + 6x + 24}$$

13. $(7x^3 - 4x^2 + 7x + 18) \div (x + 2)$

$$\begin{array}{r|rrrr} -2 & 7 & -4 & 7 & 18 \\ & \downarrow & -14 & 36 & -86 \\ \hline & 7 & -18 & 43 & -68 \end{array}$$

$$\boxed{7x^2 - 18x + 43 - \frac{68}{x+2}}$$

14. $(2x^4 - 7x^3 - 15x - 4) \div (x - 4)$

$$\begin{array}{r|rrrrr} 4 & 2 & -7 & 0 & -15 & -4 \\ & \downarrow & 8 & 4 & 16 & 4 \\ \hline & 2 & 1 & 4 & 1 & 0 \end{array}$$

$$\boxed{2x^3 + x^2 + 4x + 1}$$

Given $f(x) = 3x - 2$, $g(x) = x^2 - 4$ and $h(x) = x + 5$, find the following:

15. $(f+g)(-2) = -8 + 0 = \boxed{8}$

16. $(f-g)(6) = 16 - 32 = \boxed{-16}$

$$f(-2) = 3(-2) - 2 = -8$$

$$f(6) = 3(6) - 2 = 16$$

$$g(-2) = (-2)^2 - 4 = 0$$

$$g(6) = (6)^2 - 4 = 32$$

17. $(f \cdot g)(1) = 1 \cdot -3 = \boxed{-3}$

18. $(g/h)(0) = \frac{-4}{5}$

$$f(1) = 3(1) - 2 = 1$$

$$g(0) = (0)^2 - 4 = -4$$

$$g(1) = 1^2 - 4 = -3$$

$$h(0) = 0 + 5 = 5$$

19. $g(f(-1)) = \boxed{21}$

20. $f(g(3)) = \boxed{13}$

$$f(-1) = 3(-1) - 2 = -5$$

$$g(-5) = (-5)^2 - 4 = 21$$

$$g(3) = (3)^2 - 4 = 5$$

$$f(5) = 3(5) - 2 = 13$$

For 21-22 use $f(x) = x - 5$ and $g(x) = x^2 + 2$

21. Find $(f \circ g)(x)$

$$f(g(x))$$

$$(x^2 + 2) - 5$$

$$\boxed{x^2 - 3}$$

22. Find $(g \circ f)(x)$

$$g(f(x))$$

$$(x - 5)^2 + 2$$

$$\boxed{(x - 5)^2 + 2} =$$

Find the inverse of the following functions. $f^{-1}(x)$

23. $f(x) = 2x - 5$

$$y = 2x - 5$$

$$x = 2y - 5$$

$$+5 \quad +5$$

$$\frac{x+5}{2} = \frac{2y}{2}$$

$$\boxed{f^{-1}(x) = \frac{x+5}{2}}$$

24. $f(x) = \frac{1}{2}x + 3$

$$y = \frac{1}{2}x + 3$$

$$x = \frac{1}{2}y + 3$$

$$-3 \quad -3$$

$$2 \cdot x - 3 = 2 \cdot \frac{1}{2}y$$

$$2(x-3) = y$$

$$\boxed{f^{-1}(x) = 2(x-3) \text{ or } 2x - 6}$$

25. $y = \sqrt{x+4}$

$$x^2 = \sqrt{y+4}^2$$

$$x^2 = y + 4$$

$$-4 \quad -4$$

$$x^2 - 4 = y$$

$$\boxed{f^{-1}(x) = x^2 - 4}$$

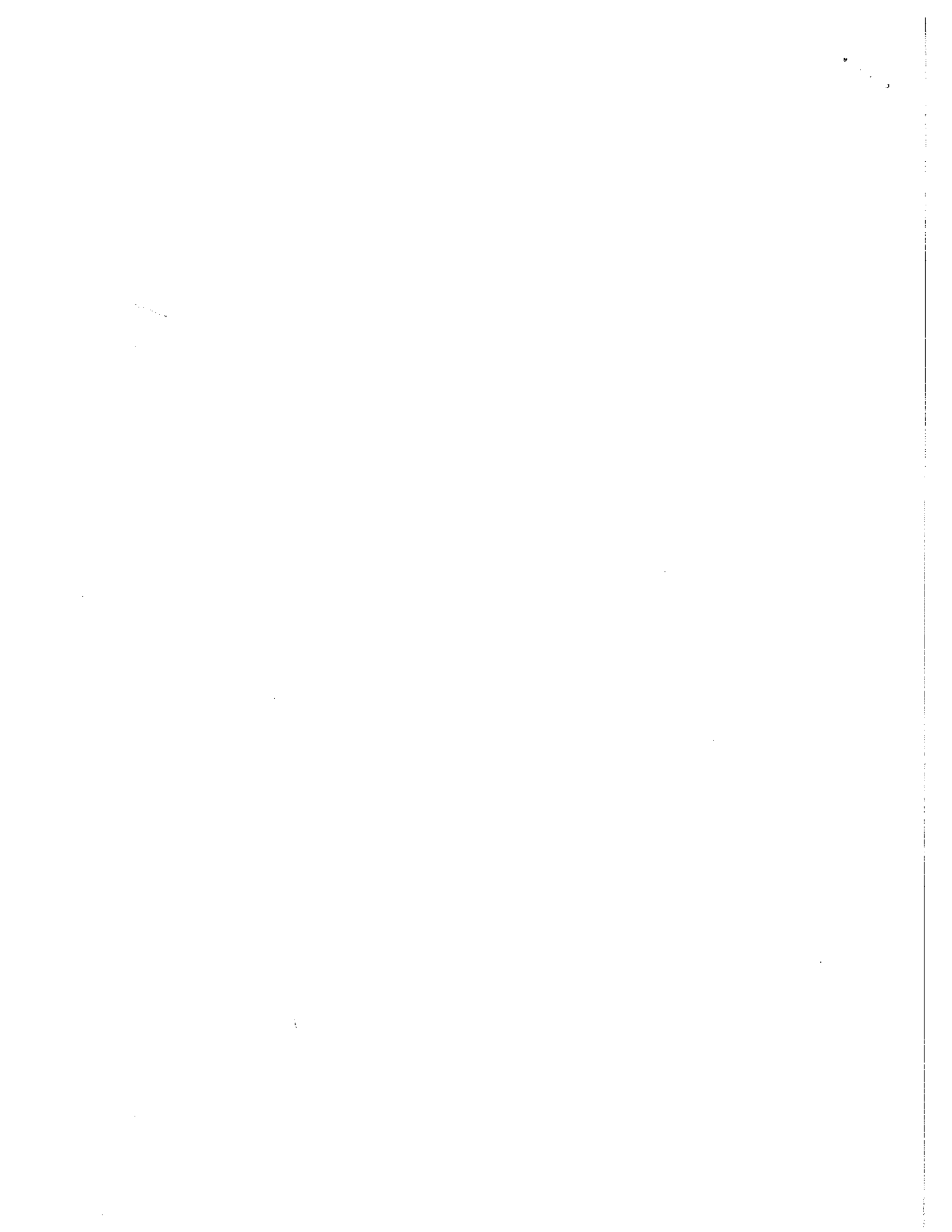
26. $y = \sqrt{x} - 3$

$$x = \sqrt{y} - 3$$

$$+3 \quad +3$$

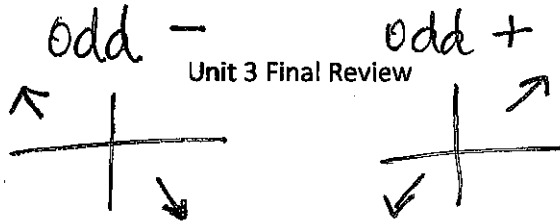
$$(x+3)^2 = \sqrt{y}^2$$

$$\boxed{f^{-1}(x) = (x+3)^2}$$



Informal Alg

Unit 3 Final Review

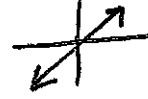


Give (1) the **MAXIMUM** number of possible turns and (2) the left and (3) right behavior of each polynomial function.

1 $y = -2x^3 + x - 1$

2 $y = x^2 + 2x - 1$

3 $y = 2x + 1$



of turns

2

1

0

$x \rightarrow \infty$ right behavior: $y \rightarrow -\infty$

$y \rightarrow +\infty$

$y \rightarrow +\infty$

$x \rightarrow -\infty$ left behavior: $y \rightarrow +\infty$

$y \rightarrow +\infty$

$y \rightarrow -\infty$

4 $y = 2x^5 - 6x^4 + 4x^3 - 2$

5 $y = (x-5)^2 - 3$

6 $y = -2x^{12} + x - 1$

of turns:

5

0

11

$x \rightarrow \infty$ right behavior: $y \rightarrow +\infty$

$y \rightarrow +\infty$

$y \rightarrow -\infty$

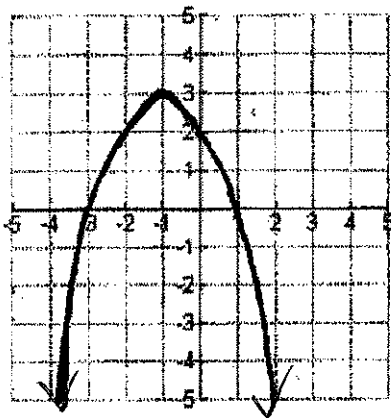
$x \rightarrow -\infty$ left behavior: $y \rightarrow +\infty$

$y \rightarrow -\infty$

$y \rightarrow -\infty$

Use the graph to find a.) where the graph is increasing or decreasing, b.) the relative minimum and relative maximum, c.) the x- and y-intercept, d) whether the LC is positive or negative and the degree is even or odd.

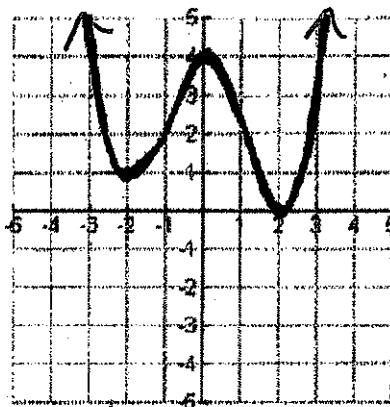
7.



Min: none
 x-int: -3, 1
 LC: neg

Max: $y = 3$
 y-int: (0, 2)
 Degree: even

8.



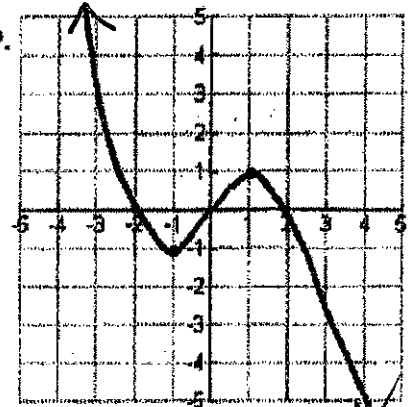
local max $y = 4$
 local min $y = 1$

abs Min: $y = 0$
 x-int: (2, 0)
 LC: pos

abs Max: none
 y-int: (0, 4)
 Degree: odd

even

9.

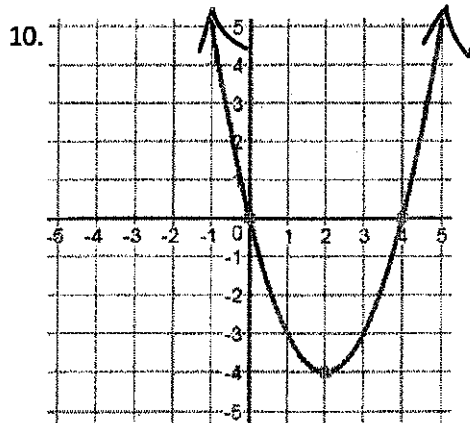


local max $y = 1$
 local min $y = -1$
 abs Min: none
 abs Max: none
 x-int: (-2, 0) (2, 0)
 y-int: (0, 0)
 LC: (0, 0)
 Degree: odd

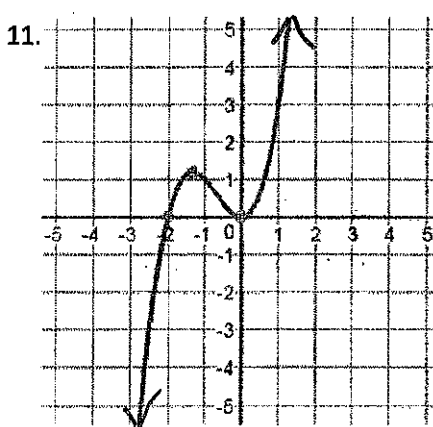
neg.

odd

Find the domain and range of each graph. (write your answer in **inequality** notation)



Domain: all \mathbb{R}
 Range: $y \geq -4$



Domain: all \mathbb{R}
 Range: all \mathbb{R}

Given the following function, use the remainder theorem to find the value.

12. $f(x) = -2x^3 + 4x^2 - 6x + 1$

a.) $f(-1)$

$$\begin{array}{r} -1 \overline{) -2 \ 4 \ -6 \ 1} \\ \underline{\downarrow \ 2 \ -6 \ 12} \\ -2 \ 6 \ -12 \ \textcircled{13} \end{array}$$

$f(-1) = 13$

b.) $f(0)$

$$\begin{array}{r} 0 \overline{) -2 \ 4 \ -6 \ 1} \\ \underline{\downarrow \ 0 \ 0 \ 0} \\ -2 \ 4 \ -6 \ \textcircled{1} \end{array}$$

$f(0) = 1$

c.) $f(2)$

$$\begin{array}{r} 2 \overline{) -2 \ 4 \ -6 \ 1} \\ \underline{\downarrow -4 \ 0 \ -12} \\ -2 \ 0 \ -6 \ \textcircled{-11} \end{array}$$

$f(2) = -11$

Determine whether the given x-value is a zero of the given function.

13. $x^4 - 8x^3 + 10x^2 + 2x + 4$, $x = 2$

$$\begin{array}{r} 2 \overline{) 1 \ -8 \ 10 \ 2 \ 4} \\ \underline{\downarrow \ 2 \ -12 \ -4 \ -4} \\ 1 \ -6 \ -2 \ -2 \ \textcircled{0} \end{array}$$

Use because
 Remainder is 0.

14. $x^5 - 25x^3 - 7x^2 - 37x - 18$, $x = -5$

$$\begin{array}{r} -5 \overline{) 1 \ 0 \ -25 \ -7 \ -37 \ -18} \\ \underline{\downarrow \ -5 \ 25 \ 0 \ 35 \ 10} \\ 1 \ -5 \ 0 \ -7 \ -2 \ \textcircled{-8} \end{array}$$

No
 because
 $R = -8$

Given the following function

a.) find the number and types of zeros

b.) find all zeros of the function (real and imaginary) given one real root

15. $f(x) = x^3 - 2x^2 + 16x - 32$, $x = 2$

a) 3 zeros ; 1 TR, 2 imag

$$\begin{array}{r|rrrr} 2 & 1 & -2 & 16 & -32 \\ & \downarrow & 2 & 0 & 32 \\ \hline & 1 & 0 & 16 & 0 \end{array}$$

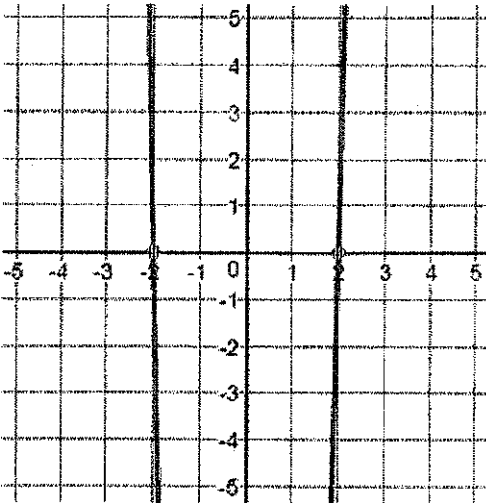
$$x^2 + 16 = 0$$

b) $x = 2$, $x^2 = -16$
 $x = \pm 4i$

a.) find the number and types of zeros

b.) find all zeros of the function (real and imaginary) given the graph

17. $f(x) = x^4 + 3x^2 - 28$



16. $f(x) = x^3 - x^2 - 18x + 8$, $x = -4$

a) 3 zeros ; 3 TR

$$\begin{array}{r|rrrr} -4 & 1 & -1 & -18 & 8 \\ & \downarrow & -4 & 20 & -8 \\ \hline & 1 & -5 & +2 & 0 \end{array}$$

$$x^2 - 5x + 2 = 0$$

use Q.F.

$$\frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(2)}}{2(1)} =$$

$$\frac{5 \pm \sqrt{17}}{2} \text{ and } 2$$

a) 4 zeros ; 2 TR, 2 imag.

$$x = \pm 2$$

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & 3 & 0 & -28 \\ & \downarrow & 2 & 4 & 14 & 28 \\ \hline & 1 & 2 & 7 & 14 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & 2 & 7 & 14 & 0 \\ & \downarrow & -2 & 0 & -14 & 0 \\ \hline & 1 & 0 & 7 & 0 & 0 \end{array}$$

$$x^2 + 7 = 0$$

$$x^2 = -7$$

$$x = \pm i\sqrt{7}$$

b) Zeros

$$x = +2, -2$$

$$+i\sqrt{7}, -i\sqrt{7}$$