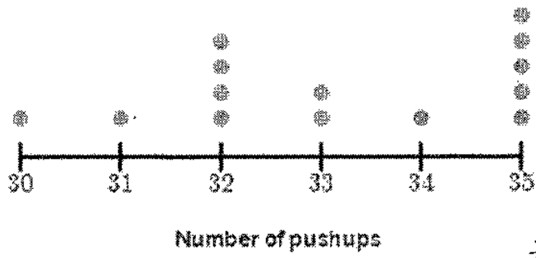
Unit 3 Review #1 Name: ____________________ Date: __________

Section I – Percentiles

Use the dot plot below to answer the questions.



*percentile

$\frac{\text{\# less than or equal}}{\text{total}}$

1. Joseph could do 33 pushups in a minute. What is their percentile?

$\frac{8}{14} = 0.57$ $\boxed{57^{th} \text{ percentile}}$

2. Patricia could do 31 pushups in a minute. What is their percentile?

$\frac{2}{14} = 0.14$ $\boxed{14^{th} \text{ percentile}}$

3. Roderick could do 35 pushups in a minute. What is their percentile?

$\frac{14}{14} = 1$ $\boxed{99^{th} \text{ percentile}}$ (never 100^{th} percentile)

Section II – Solve for Z-Scores

4. Layla scored a 65 on her Spanish final exam. What was her z-score if the average on the test was a 69 and the standard deviation was 4?

$Z = \frac{X - \bar{X}}{Sx}$ $Z = \frac{65-69}{4} = \boxed{-1}$

*score was one st.deviation below the mean.

5. Gabriela scored a 76 on her Statistics final exam. What was her z-score if the average on the test was an 84 and the standard deviation was 7?

$Z = \frac{76-84}{7} = \boxed{-1.14}$

*score was 1.14 st. deviations below the mean

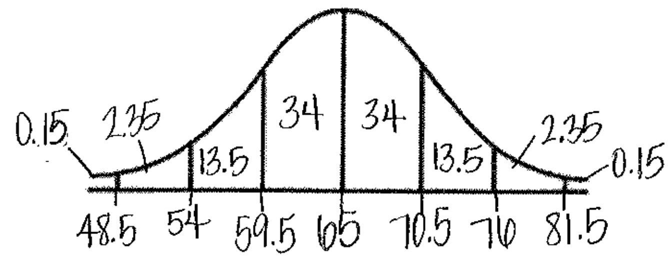
6. Between Layla and Gabriela, who did relatively better on their final exam? Why?

Layla, she has a higher Z-score.

*negative #s are higher if they are closer to zero

Section III – Empirical Rule

7. The Unit 2 Statistics test had an average of 65 after 55 students took the test. Label the normal distribution if the $\sigma = 5.5$.



a. What percentage of scores were between 59.5 and 70.5?

$34+34=\boxed{68\%}$

b. What percentage of scores were outside of 48.5 and 81.5?

$0.15+0.15=\boxed{0.3\%}$

c. What percentage of scores were less than 59.5?

$13.5+2.35+0.15=\boxed{16\%}$

d. What percentage of scores were between 59.5 and 81.5?

$34+34+13.5+2.35=\boxed{83.85\%}$

e. What percentage of scores were between 48.5 and 59.5?

$2.35+13.5=\boxed{15.85\%}$

f. What percentage of scores were between 65 and 76?

$34+13.5=\boxed{47.5\%}$

g. **How many** students made below a 65?

% (total) = 50% • 55 $\quad 0.50(55)=\boxed{27.5 \text{ students}}$

h. **How many** students made above an 81.5?

$0.15\% = 0.0015(55)=\boxed{0.0825 \text{ students}}$ (not possible)

i. What score separated the top 16%?

start at the top and add percents until you get 16%

$0.15+2.35+13.5=16\% \rightarrow \boxed{70.5}$

j. What score separated the bottom 2.5%?

$0.15+2.35=2.5\% \rightarrow \boxed{54}$

Section IV – Using the Z-table (Easy)

For the numbers below, find the percentile rank (two decimal places) (percent of individuals scoring **BELOW**):

8. z = 0.24

0.5948 $\quad \boxed{59.48\%}$

9. z = -1.25

0.1056 $\quad \boxed{10.56\%}$

10. z = 0.08

0.5319 $\quad \boxed{53.19\%}$

11. z = -0.47

0.3192 $\boxed{31.92\%}$

12. z = 3.2

0.9993 $\boxed{99.93\%}$

13. z = -2.3

0.0107 $\boxed{1.07\%}$

14. A fifth grader takes a standardized achievement test ($\mu = 125$ and $\sigma = 15$) and scores a 133. What is the child's percentile rank?

$$Z = \frac{133-125}{15} = 0.53$$

tbl = 0.7019

$\boxed{70.19\%}$

Section V – Using the Z-table (Medium)

For the numbers below, find the percent of cases falling **ABOVE** the z-score:

(subtract from 1

15. z = 0.24

0.5948 1-0.5948=0.4052 $\boxed{40.52\%}$

16. z = -1.25

0.1056 1-0.1056=0.8944 $\boxed{89.44\%}$

17. z = 0.08

0.5319 1-0.5319 = 0.4681 $\boxed{46.81\%}$

18. A patient recently diagnosed with Alzheimer's disease takes a cognitive ability test and scores a 51. The mean on the test is 52 and has a standard deviation of 5. What percentage of people scored **higher** on the cognitive test?

$$Z = \frac{51-52}{5} = -0.2$$

tbl = 0.4207

1-0.4207= 0.5793 $\boxed{57.93\%}$

For the numbers below, find the percent of cases falling **BETWEEN** the z-score:

19. -0.32 < z < -0.23

0.3745 0.4090

0.4090-0.3745=0.0345 $\boxed{3.45\%}$

20. 0.03 < z < 2.7

0.5120, 0.9985

0.9985-0.5120 =0.4845 $\boxed{48.45\%}$

21. -1.4 < z < 1.84

0.0808 0.9671

0.9671-0.0808 = 0.8863 $\boxed{88.63\%}$

22. Pat and Chris both took a spatial abilities test (mean = 80, std. dev. = 8). Pat scores a 76 and Chris scored a 94. What percent of individuals scored between Pat and Chris?

$\boxed{\text{Pat}}$ $Z = \frac{76-80}{8} = -0.5$

tbl = 0.3085

$\boxed{\text{Chris}}$ $Z = \frac{94-80}{8} = 1.75$

tbl = 0.9599

0.9599-0.3085 = 0.6514 $\boxed{65.14\%}$

23. The Welcher Adult Intelligence Test Scale is composed of a number of subtests. On one subtest, the raw scores have a mean of 35 and a standard deviation of 6. Assuming these raw scores form a normal distribution:

a. What is the probability of getting a raw score between 28 and 38?

① $Z = \frac{28-35}{6} = -1.17$ ② $Z = \frac{38-35}{6} = 0.5$ $0.6915 - 0.1210$

tbl $= 0.1210$ tbl $= 0.6915$ 0.5705

$\boxed{57.05\%}$

b. What is the probability of getting a raw score between 41 and 44?

① $Z = \frac{41-35}{6} = 1$ tbl $= 0.8413$ ② $Z = \frac{44-35}{6} = 1.5$ tbl $= 0.9332$

$0.9332 - 0.8413$

0.0919

$\boxed{9.19\%}$

Section VI – Using the Z-Table Reverse

24. Find the z-score that gives a probability of 0.2810.

$\boxed{Z = -0.58}$

25. Find the z-score that give the area above 0.1515.

$1 - 0.1515 = 0.8485$ $\boxed{Z = 1.03}$

26. For a normal distribution, find the z-score that separates the distribution as follows:

a. Separate the **highest** 27% from the rest of the distribution.

① $27\% = 0.27$ ③ $0.73 - 0.7291 = 0.0009$ * $\boxed{Z = 0.61}$

② $1 - 0.27 = 0.73$ $0.73 - 0.7324 = -0.0024$

b. Separate the **lowest** 42% from the rest of the distribution.

① $42\% = 0.42$ ② $0.42 - 0.4207 = 0.0007$ *

$0.42 - 0.4168 = 0.0032$ $\boxed{Z = -0.20}$

c. Separate the **highest** 70% from the rest of the distribution.

① $70\% = 0.7$ ② $0.3 - 0.3015 = 0.0015$ *

② $1 - 0.7 = 0.3$ $0.3 - 0.2981 = 0.0019$ $\boxed{Z = -0.52}$

d. Separate the **lowest** 89% from the rest of the distribution.

① $89\% = 0.89$ ② $0.89 - 0.8888 = 0.0012$ $\boxed{Z = 1.23}$

$0.89 - 0.8907 = 0.0007$ *

Section VII – Solving for the observation (x-value) HINT: Z is given, set up equation and solve for x.

27. Sam took the ACT and his score was one standard deviation (z=1) above the average. If the ACT has a mean of 20.8 and a standard deviation of 4.8, what was Sam's score?

$4.8 \cdot 1 = \frac{X - 20.8}{4.8} \cdot 4.8$

$4.8 = X - 20.8$

$+20.8 \quad +20.8$

$\boxed{X = 25.6}$

28. Jimmy took the SAT and his Math section score was two standard deviations below (z=-2) the average. If the Math section of the SAT has an average of 533 and a standard deviation of 100, what was Jimmy's math section score?

$$100 \cdot -2 = \frac{X-533}{100} \cdot 100 \rightarrow -200 = X-533$$

$$+533 \quad +533$$

$$\boxed{333 = X}$$

Section VIII – Solving for the observation (x-value) HINT: Use the table first to find the z-score, then solve for x.

29. The Welcher Adult Intelligence Test Scale is composed of a number of subtests. On one subtest, the raw scores have a mean of 35 and a standard deviation of 6. Assuming these raw scores form a normal distribution:

a) What number represents the 70th percentile (what number separates the lower 70% of the distribution)?

1. $70\% = 0.70$
2. $0.7 - 0.6985 = 0.0015*$
   $0.7 - 0.7019 = 0.0019$
3. $z = 0.52$

$$6 \cdot 0.52 = \frac{X-35}{6} \cdot 6$$

$$3.12 = X-35$$

$$+35 \quad +35$$

$$\boxed{38.12 = X}$$

b) What number represents the 99.83rd percentile?

1. $99.83\% = 0.9983$
2. $z = 2.93$

$$6 \cdot 2.93 = \frac{X-35}{6} \cdot 6$$

$$17.53 = X-35$$

$$+35 \quad +35$$

$$\boxed{52.58 = X}$$

30. Scores on the SAT form a normal distribution with $\mu = 500$ and $\sigma = 100$.

a) What is the minimum score necessary to be in the **bottom** 17% of the SAT distribution?

1. $17\% = 0.17$
2. $0.17 - 0.1711 = 0.0011*$
   $0.17 - 0.1685 = 0.0015$
3. $z = -0.95$

$$100 \cdot -0.95 = \frac{X-500}{100} \cdot 100 \rightarrow -95 = X-500$$

$$+500 \quad +500$$

$$\boxed{405 = X}$$

b) Find the range of values that defines the **top** 40% of the distribution of SAT scores.

1. $40\% = 0.4$
2. $1 - 0.4 = 0.6$
3. $0.6 - 0.5987 = 0.0013*$
   $0.6 - 0.6026 = 0.0026$
4. $z = 0.25$

$$0.25 = \frac{X-500}{100}$$

$$25 = X-500$$

$$\boxed{525 = X}$$

31. If a math test scores were normally distributed with a mean of 81 and a standard deviation of 5, what score is in the 90th percentile?

① 90=0.90

② 0.90 − 0.8997 = 0.0003 *

0.90 − 0.9015 = 0.0015

③ Z = 1.28

$$1.28 = \frac{X-81}{5}$$

6.4 = X − 81

$\boxed{X = 87.4}$

32. If a Math test scores were normally distributed with a mean of 79 and a standard deviation of 7, what score is in the 23rd percentile?

① 23% = 0.23

② 0.23 = 0.2327 = 0.0027

0.23 = 0.2296 = 0.0004*

③ Z = −0.74

$$-0.74 = \frac{X-79}{7}$$

−5.18 = X − 79

$\boxed{X = 73.82}$

33. If a Biology test scores were normally distributed with a mean of 67 and standard deviation of 3, what score had a probability of 89.44%?

① 89.44% = 0.8944

② Z = 1.25

$$1.25 = \frac{X-67}{3}$$

3.75 = X − 67

$\boxed{70.75 = X}$

34. If a factory created bolts that lengths followed a normally distribution with a mean of 3.5 inches and a standard deviation of 0.2 inches, what bolt length would be in the **bottom** 0.41%?

① 0.41% = 0.0041

② −2.64 = Z

$$-2.64 = \frac{X-3.5}{0.2}$$

−0.528 = X − 3.5

$\boxed{2.972 = X}$

35. Matthew scored in the 94.52nd percentile on his IQ test which has an average of 110 and $\sigma = 20$. What did he score on his IQ test?

① 94.52% = 0.9452

② Z = 1.60

$$1.60 = \frac{X-110}{20}$$

32 = X − 110

$\boxed{X = 142}$